A honore di tucti magistri et scolari de questa scienza. Et de qualunqua altra bona persona vedesse et legesse questo tractato devoto e ragionevolmente

and in particular in memoriam

Joseph Needham
Nikolay Bukharin
Edgar Zilsel
Kurt Vogel
Gino Arrighi

Preface

In 1984, I was present when Joseph Needham gave a dinner talk at the Sarton Centennial Conference. During or just after the talk I whispered to the colleague sitting next to me "He does not know, but he was my only teacher in the history of science". Back whispered Samuel Edgerton – he, indeed, was the neighbour, and I hope he will forgive me for telling – "He was mine, too".

One of the things I learned from Joseph Needham's *Grand Titration* [1969] – recently published at the time my serious interest in the history of science began, and the first thing from his hand I read – was to take the relation between "clerks and craftsmen" seriously as a complicated interplay. Needham, of course, made no secret of having received inspiration both from Edgar Zilsel and from the Soviet contributions to the legendary 1931-conference on the history of science. When reading these, what impressed me as intellectually most sophisticated was Nikolay Bukharin's London-paper [1931/1971].[1]

My receptivity to Needham's and Bukharin's interest in the intricate relations between theoreticians' and practitioners' knowledge was certainly enhanced by my own experience as a physics teacher in an engineering school, where I was confronted time and again with the inability of university-trained physicists and mathematicians to create a bridge between the shape of their own knowledge and the interests, orientation and knowledge of students who thought about building houses and bridges.

As I was also engaged at the times in the Danish debate about the "new math" reform, my general interest in the history of science concentrated naturally on the history of

[1] I thus agree with I. Bernard Cohen's words as quoted by Loren Graham [1993: 241] that "Bukharin's piece remains impressive today to a degree that Hessen's is not".

mathematics as soon as I started doing my own work in the field – and, as it turned out, mostly on the history of pre-Modern mathematics. Along with other topics I maintained my interest in practitioners' knowledge as it could be encountered in the sources – which, when we speak of pre-Modern mathematical practice, was often only possible through teaching material directed at future practitioners or otherwise reflecting practitioners' knowledge. Though for a long time I did not work directly on late medieval European vernacular mathematics, I collected material when I encountered it – in particular source editions. Much I found in Baldassare Boncompagni's marvellous *Bullettino*, much I owe to Moritz Cantor and the circle around him. I also had the good luck to come in personal touch with Kurt Vogel and Gino Arrighi and to receive directly from them publications which even my excellent interlibrary service would have been unable to get hold of.

Two accidents pushed me to capitalize on this material. First, in November 1996, at a meeting at the Mathematisches Forschungsinstitut Oberwolfach, Henk Bos asked me to improvise on 43 hours notice a presentation of what had happened in the historiography of European medieval mathematics during the last forty years and to summarize the picture which now emerged. I had no support at hand beyond my memory, my personal catalogue and my own writings from the last decade, catalogue and writings as present on my laptop. None the less I accepted, maybe because of my personal affection for the organizers, maybe because I felt as absurdly flattered as the father asked by his kid to repeat a magnificent sunset – maybe for still other reasons. While working on my talk I discovered structures in the story which I had not been conscious of knowing about; during the next months I therefore set myself the goal to do the work as it *should* have been done, looking once more at relevant sources and publications (which for a project of this kind were also primary sources), sifting and ordering the material, etc.

The next accident was that I reread for this purpose an article which Louis Karpinski had published in 1929 about "The Italian Arithmetic and Algebra of Master Jacob of Florence, 1307". Knowing now Arabic algebra much better than I had when first reading the paper in 1977, I came to suspect that Jacopo's treatise might have astounding implications for our understanding of the origins of European vernacular algebra (the reasons for this are explained in detail in the Introduction, p. 3). The suspicion had to be verified, and I shelved my medieval survey (it remains shelved) and got hold of the manuscript Karpinski had described. Working first on the algebra section alone, afterwards on the rest of the treatise and other early abbacus writings – that is, mathematical writings made in the context of the late medieval and Renaissance *scuole d'abbaco* for merchant youth – I discovered that much more had to be changed in the conventional picture than I had at first suspected.

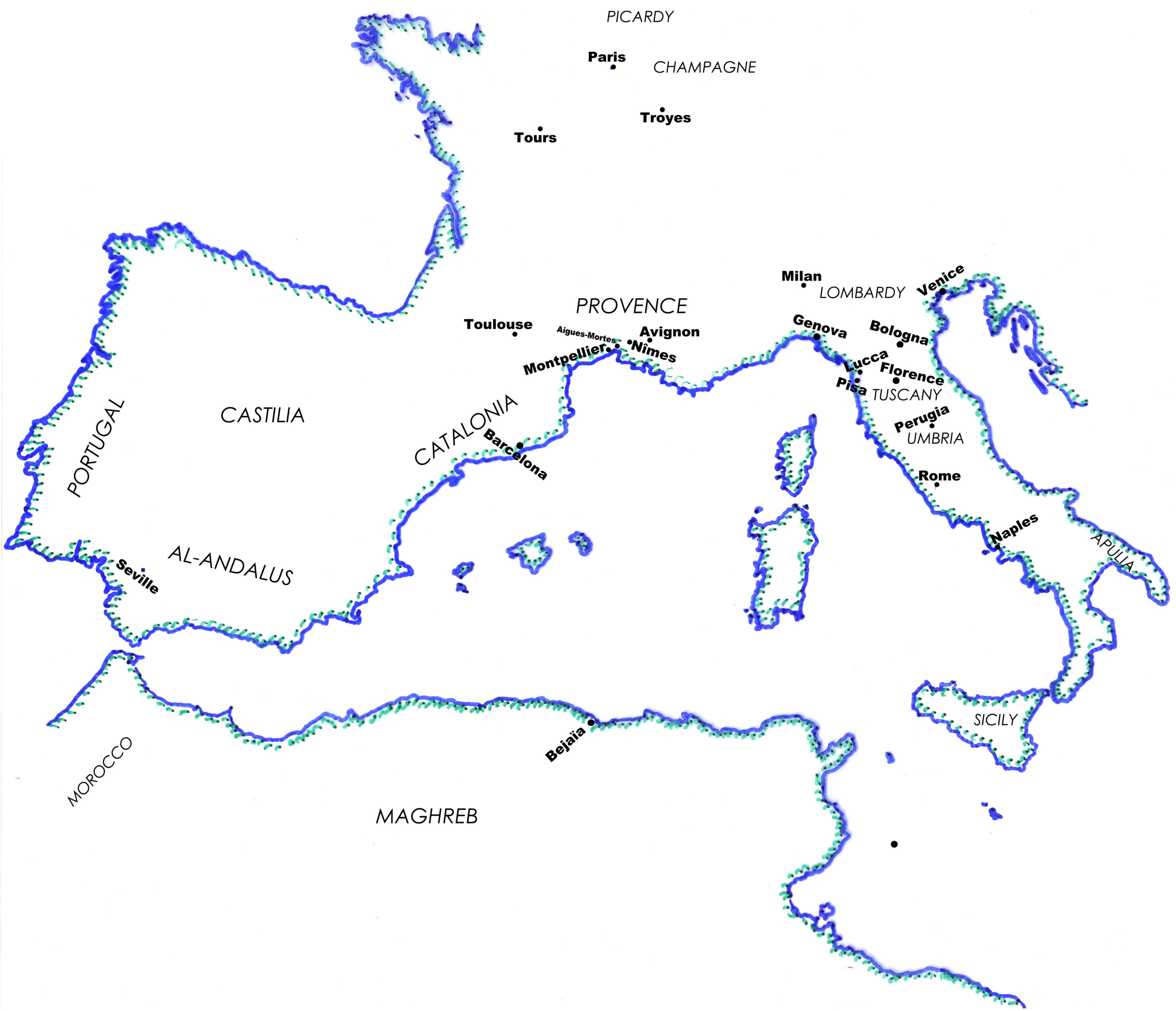
Much of abbacus mathematics was still around when I went to primary and early secondary school in the 1950s (that was before the arrival of the "new math" and the reactions to it, which to some extent changed the situation). Abbacus mathematics itself, however, can be known only by those who read Italian, since no editions of the manuscripts are accompanied by translations. I therefore realized that any challenge to the conventional picture had to be based on a text edition accompanied by a translation and copious references to other parts of the record.

I also thought that the challenge *had* to be made. According to the conventional picture, abbacus mathematics is a simplified version of a *great book* written in Latin (Leonardo Fibonacci's *Liber abbaci*), adapted by less able teachers to a school with rather modest pretensions. Close scrutiny of the sources shows that interactions between different cultures at the practitioners' own level was much more important than the "great book". In a certain way, the present book thus pays a debt to the manes of Joseph Needham, Nikolay Bukharin and Edgar Zilsel.

So much about what induced me to work on the history of abbacus mathematics and to write the present book. About the book itself I shall only say that it falls into two parts of roughly equal length, one examining Jacopo's treatise together with the whole context of early abbacus mathematics, the other containing an edition of the treatise itself (made from the Vatican manuscript Vat. Lat. 4826) accompanied by a very literal translation; an appendix contains a "semi-critical" edition of a revised version of the treatise, made from Milan, Trivulziana MS 90, collated with Annalisa Simi's transcription of Florence, Riccardiana MS 2236. Details are better read from the table of contents. Indexes of names, subjects and sources referred to should facilitate the use of the book.

References are made according to the author-date system (in the case of text editions, the editor takes the place of the author). All translations into English are mine if nothing else is stated.

For many years I have been in helpful contact with Menso Folkerts, Ivo Schneider, Ulrich Rebstock, Jacques Sesiano, Barnabas Hughes, Tony Lévy and Ahmed Djebbar. Specifically for my work on abbacus matters this last decade I have also benefitted from exchanges with Warren Van Egmond, Mahdi Abdeljaouad, Wolfgang Kaunzner, Charles Burnett, Raffaella Franci, Laura Toti Rigatelli, Annalisa Simi, Enrico Giusti, Lucia Travaini, Maryvonne Spiesser, Stéphane Lamassé, Betsabé Caunedo del Potro and Maria do Céu Silva, and occasionally with many other colleagues. It is a pleasant duty to thank all of them for assistance, inspiration and challenges.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THE VATICAN MANUSCRIPT EDITION AND TRANSLATION

JACOPO, HIS TREATISE, AND ABBACUS CULTURE

Introduction

In [1929a], Louis Karpinski published a short description of "The Italian Arithmetic and Algebra of Master Jacob of Florence, 1307". Around one third of the note describes the algebra chapter of the treatise, pointing out among other things that Jacopo presents the algebraic "cases" in a different order than al-Khwārizmī, Abū Kāmil, and Leonardo Fibonacci, and that the examples that follow the rules also differ from those of the same predecessors. Karpinski did not mention explicitly the absence of geometric proofs of the rules, nor that the examples differ from those of the other authors already in general style, not only in detailed contents; but attentive reading of Karpinski's text and excerpts from the manuscript leave little doubt on either account.

In retrospect, Karpinski's succinct article should therefore have been a challenge to conventional thinking, both about the history of pre-Renaissance algebra and about applied arithmetic in general from the same period. Nonetheless, I have not been able to discover any echo whatsoever of his publication. This may have at least three reasons.

Firstly, 1929 fell in a period where the interest in European medieval mathematics was at a low ebb – probably the lowest since the Middle Ages, at least since 1840. From 1920 to c. 1948 (from Moritz Cantor's death to the beginning of Marshall Clagett's work in the field), the total number of scholarly publications dealing with European medieval mathematics (Latin as well as vernacular) does not go much beyond the dozen.

Secondly, the existence of the distinct *abbaco*[2] mathematical tradition was not recognized, although Karpinski had already described another abbacus treatise in [1910], and Italian nineteenth-century local history had dealt with the *scuole d'abbaco* of many localities. As early as [1900: 166], it is true, Cantor had spoken of the existence throughout the fourteenth century of two coexisting "schools" of mathematics, one "geistlich" ("clerical", that is, universitarian), the other "weltlich oder kaufmännisch" ("secular or commercial", supposedly derived from Leonardo Fibonacci's work). Part of Cantor's basis for this was Guglielmo Libri's edition [1838: III, 302–349] of a major section of what has now been identified beyond reasonable doubt as Piero della Francesca's *Trattato*

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[2] The late medieval sources alternate between the spellings *abaco* and *abbaco*. In order to avoid confusion with the calculation-board or -frame I shall stick to the spelling *abbaco* and the Anglo-Latin analogue *abbacus* (except of course in quotations, which follow the source that is quoted).

As formulated by Richard Goldthwaite [1972: 419], "Originally the term *abbaco* referred to a device for calculating by means of disks, beads, counters, etc.; but in Italy after the introduction of the use of the Arabic numerals [...], the term *abbaco* came to be used in a general sense for instruments, methods, manuals, schools, teachers or anything else related to the skill of doing computations, especially with reference to practical applications in the mercantile world".

d'abaco[3] (which Cantor, accepting Libri's wrong dating, had located in the fourteenth century); also known to Cantor was the material collected and published by Baldassare Boncompagni, for instance in [1854], which, as a matter of fact, was already a sufficient basis for the claim. Georg Eneström [1906] had done what he could to make a fool of Cantor by twisting his words.[4] Sensitive reading should have exposed Eneström's arrogant fraud; but the kind of knowledge that was required for that had come to be deemed irrelevant for historians of mathematics and hence forgotten, and George Sarton [1931: 612*f*] not only cites Eneström's article but embraces the whole thesis uncritically.

Thirdly, like Cantor, Karpinski took the continuity from Fibonacci onward for granted, and concluded [1929a: 177] that the

> treatise by Jacob of Florence, like the similar arithmetic of Calandri, marks little advance on the arithmetic and algebra of Leonard of Pisa. The work indicates the type of problems which continued current in Italy during the thirteenth to the fifteenth and even sixteenth centuries, stimulating abler students than this Jacob to researches which bore fruit in the sixteenth century in the achievements of Scipione del Ferro, Ferrari, Tartaglia, Cardan and Bombelli.

Only those interested in manifestations of mathematical stagnation – thus Karpinski's suggestion – would gain anything from looking deeper into Jacopo's treatise.

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[3] On the identification of Libri's manuscript with the very manuscript from which Arrighi [1970a] made his edition, see [Davis 1977: 22*f*]. Margaret Davis also gives the argument that the anonymous treatise must indeed be ascribed to Piero.

[4] Arguing from his own blunt ignorance of the institution within which university mathematicians moved, Eneström rejected the epithet "clerical" as absurd ("Sacrobosco und Dominicus Clavasio waren meines Wissens nicht Geistliche"; actually, all university scholars were at least in lower holy orders, as evident from the familiar fact that they were submitted to canonical jurisdiction). He was further convinced that merchants' mathematics teaching would never treat useless problems like the "100 fowls", and claimed that Fibonacci was only spoken of as a merchant in late and unreliable sources; therefore no "commercial" school could have been inspired by Fibonacci and teach such useless problems. Actually, Cantor had not argued from Fibonacci's profession, although he does refer to him elsewhere in pseudo-poetical allusions as the "learned merchant" – pp. 85*f*, 154; yet in the very preface to the *Liber abbaci* Fibonacci refers to what can hardly be anything but business travelling – in Boncompagni's edition [1857: 1] Fibonacci speaks of his travels to "places of business". What Eneström could not know is that all other manuscripts speak of travelling "for reasons of business" [Grimm 1976: 101*f*].